

VALUE AT RISK (VaR)

參考書目：[Modern portfolio theory and investment analysis \(2014\), 9th Edition, Wiley.](#)

Institutions such as banks and insurance companies are concerned with the likelihood of bad outcomes. A widely used approach is value at risk (Jorion, 2006). Value at risk looks only at the size of bad outcomes that can occur with a specified probability in a specific time interval.

金融機構非常關心發生不良結果（損失）的可能性。一種廣泛使用的方法是風險值 (Jorion, 2006)，而風險值只關心在特定時間區間內以特定機率發生的不良結果的大小。

For example, the institution might calculate that there is a 5% probability of a loss of \$295,000 or more occurring in the next week. If management were interested in the 5% probability level, then \$295,000 would be the value at risk. Let us discuss how this value at risk is determined.

例如，某一機構可能計算出下週發生 295,000 美元或更多損失的機率為 5%。如果管理層接受這 5% 的機率水準，那麼 295,000 美元就是風險值。讓我們討論如何決定此風險值。

Assume a portfolio is \$100 million in value. Assume the expected return over the next week is 0.2%, with a standard deviation of 0.3%. Also assume normal distributions. Then we know that the lowest 5% of possible returns are returns that occur more than 1.65 standard deviations away from the mean. Thus, 5% of the time, we can expect returns below $\bar{R} - 1.65\sigma$ or $0.2 - (1.65) \times (0.3)$. Simplifying this results in a return of -0.295% or less.

假設一個投資組合的價值為 1 億美元，而下週的預期報酬率為 0.2%，標準差為 0.3%，而且假設報酬率服從常態分佈。然後我們知道最低機率 5% 的可能報酬是與平均值相差超過 1.65 個標準差的報酬率。因此，在 5% 的情況下，我們可以預期報酬率低於 $\bar{R} - 1.65\sigma$ 或 $0.2 - (1.65) \times (0.3) = -0.295\%$ 更多。具體數學定義：

$$\Pr(r_{t+1} < Z_\alpha \times \sigma_{t+1} | \Omega_t) = \alpha$$

$$\text{VaR}_{t+1} | \Omega_t = -(\hat{r}_{t+1} - Z_\alpha \times \hat{\sigma}_{t+1}) \times \text{Asset value}$$

If this investor has 100 million in assets, this is a loss of 295,000 or more. This dollar number, \$295,000, is called value at risk (VaR). VaR is the best outcome that can occur if returns are in the worst part of the possible outcomes.

如果這個投資者持有 1 億資產，這就是 29.5 萬美元或更多的損失。這個 29.5 萬美元的數字被稱為風險值 (VaR)。如果報酬是處於可能結果之中最差的部分時，VaR 會是發生最差部分的最佳結果。

Estimate the mean return and standard deviation over the period in question and use the normal distribution to determine how many standard deviations from the mean you are concerned with. The worst 5% is the common choice, which, as we discussed

earlier, is 1.65 standard deviations from the mean. This return is then computed (in the example, mean minus 1.65 standard deviations is computed) and multiplied times the value of the assets to get the least dollar loss if returns are in the worst possible set of outcomes (in our example, the lowest 5%). This is how one finds the VaR.

估計所討論的期間之平均報酬率和標準差，並使用常態分佈來決定與你所關心的平均值距離多少個標準差。最差的是 5% 是常見的選擇，正如我們之前討論的，它是偏離平均值 1.65 個標準。然後計算這個報酬率（在本例中，平均值減去 1.65 個標準差）並乘以資產價值，而得到在發生最壞的結果中（在本例中，最低的 5% 機率）最少的損失。這個最少的損失就是 VaR。

Many institutions hold assets that do not have normal distributions of returns, such as securities with option-like elements. These institutions usually use simulation to compute VaR. These institutions simulate possible return paths thousands of times and then determine the best returns among the bad outcomes. In our example, if the institution performed 1,000 simulations, and they were worried about the worst 5% of outcomes, they would sort the outcomes and, from the 50 worst outcomes (lowest 5%), take the highest return. This, times the assets, results in the dollar loss, and this dollar loss would be designated as the VaR.

許多機構持有的資產報酬率並不呈現常態分佈，例如：具有類似期權性質的證券。這些機構通常使用模擬法來計算風險值。這些機構模擬可能的報酬路徑數以千次，然後在所有的不良結果中決定最好的報酬率。在本例中，如果該機構進行了 1,000 次模擬，並且他們擔心的是 5% 的最差結果，他們會對結果進行排序，並從 50 個最差的結果（即結果最低的 5%），取出最高的報酬率，將其報酬率乘以資產價值，產生美元損失，這個美元損失就是 VaR。